The dependence structure of daily UK hydrological time series

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Introduction

The presence of dependence structure in hydrological time series is both problematic (it presents the use of standard statistical tools for analysis) and useful (as it indicates a pattern in the data that may be predicted by a model). The dependence structure determines several key characteristics of the time series all of which relate to the way in which the structure contributes to the variance as a function of the autocorrelation function:

\[
\rho(h) = \sum_{k=0}^{\infty} \rho(k) E[(x(t+h) - \mu)(x(t) - \mu)]
\]

The dependence structure tends to inflate the variance, as individual measurements don’t provide unique information. This is represented by the variance inflation factor:

\[
\frac{\text{variance}}{\text{variance without dependence}} = \frac{1}{1 - \rho}
\]

We use an analytical model to represent the dependence structure (non-randomness) in hydrological time series from the UK. We can then identify the variance of these series to mainly comprise four distinct components: quick, slow, seasonal and random noise.

This analytical model allows us to:
1. Project the distribution of totals at larger timescales
2. Identify change-points to a level of probability
3. Sensitive trend detection and attribution
4. Define a time-domain alternative to the power spectrum


Analytical model of the dependence structure

The variance inflation factor calculated from:

\[
\rho(h) = \frac{\text{variance}}{\text{variance without dependence}} = \frac{1}{1 - \rho}
\]

Forecast long-term total distributions

Example: Wheat Yield

Standard experimental design

Treatment in control

Randomized bloc design

These experiments are monitored using integrated indicators of responses to the treatments:

The value of integrated hydrological and biogeochemical indicators

Experiments are designed to find which treatments lead to statistically significant differences in outcomes

Example: River Thames Streamflow 1883 to 2015

Analytical model fitted to the autocorrelation of the Thames streamflow data:

The contribution of different component frequencies to the variance is usually determined using the power spectral density. Here we can use our data to produce this, and produce a version predicted by our analytical model:

And we can do the equivalent in the time domain using our new approach:

Application for flow decomposition

The three flow components in UK river systems...

These experiments are monitored using integrated indicators of responses to the treatments:

Integrated indicators tend to smooth out variability and amplify the mean. In the presence of short memory, they also lead to the variance in the integrated indicator being normally-distributed which makes classical statistics more readily applicable.

Simple numerical tests can show how hydrological processes can be considered as integrated indicators, which may enable more sensitive measures of trend detection.

Results show that, although there has been change, there is only a small probability of this being significant, less than required by standard statistical tests for change.